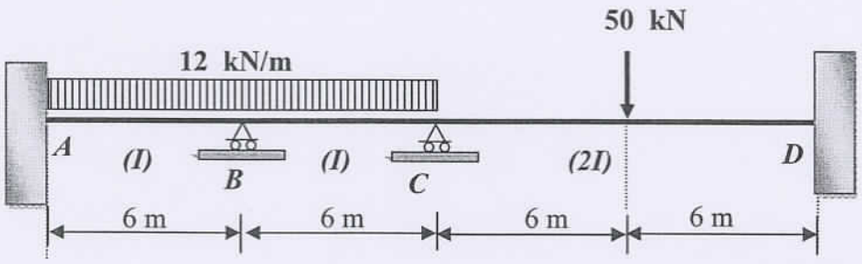
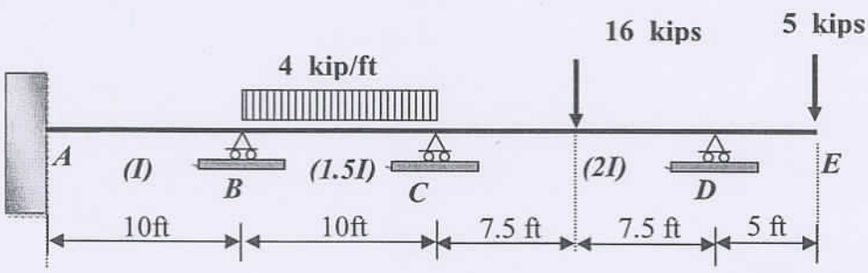
Analyze the following structures by the Slope-Deflection Method:

Problem [1]



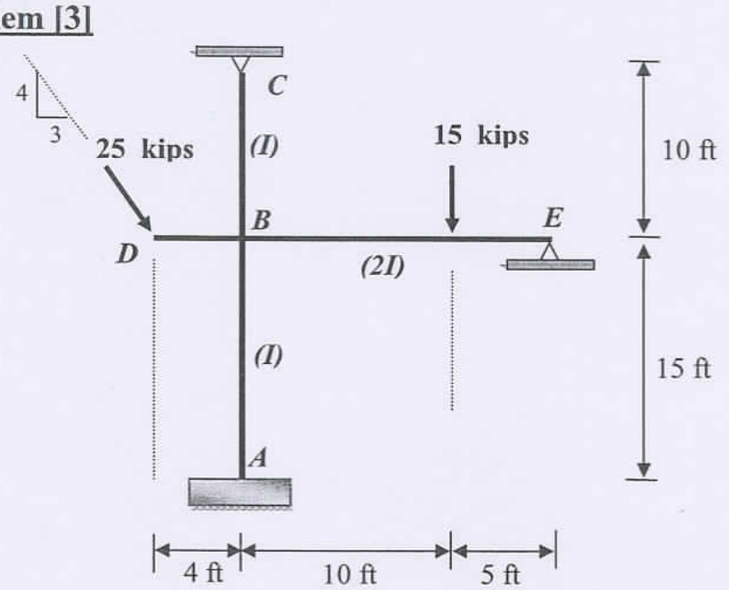
E= constant

Problem [2]



E= constant

Problem [3]

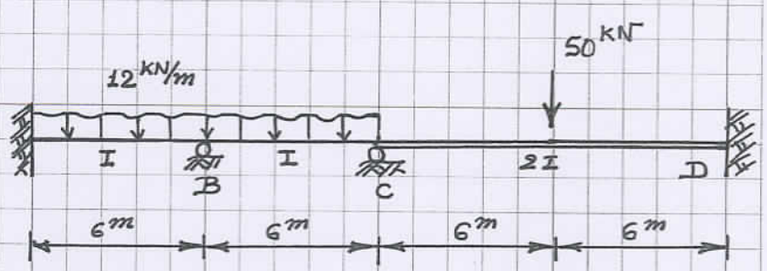


E= constant

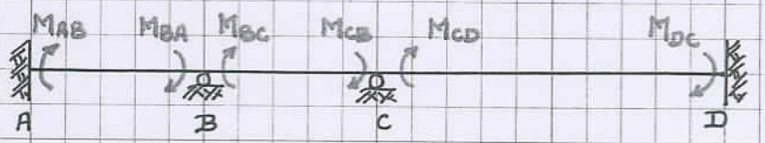
Good Luck & Best Wishes
Dr. Hisham S. Basha, Ph.D.

PROBLEM [1].

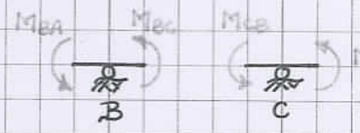
E = constant.



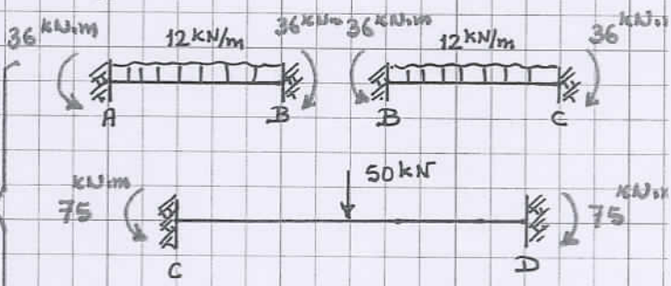
Connecting Moments
~ assumed clockwise ~



Joint Equilibrium



$\Sigma F_y = 0$



$\frac{wl^2}{12} = 36$ kN.m.

$\frac{Pl}{8} = 75$ kN.m.

* $M_{AB} = \frac{2EI}{6}\left[0 + \theta_B - 0\right] + (-36)$

$M_{AB} = \frac{EI}{3}\theta_B - 36$ .... (1)

* $M_{BA} = \frac{2EI}{6}\left[2\theta_B + 0 - 0\right] + 36$

$M_{BA} = \frac{2EI}{3}\theta_B + 36$ .... (2)

* $M_{BC} = \frac{2EI}{6}\left[2\theta_B + \theta_C - 0\right] - 36$

$M_{BC} = \frac{2EI}{3}\theta_B + \frac{\theta_C \cdot EI}{3} - 36$ ... (3)

* $M_{CB} = \frac{2EI}{6}\left[2\theta_C + \theta_B - 0\right] + 36$

$M_{CB} = \frac{2EI}{3}\theta_C + \frac{EI}{3}\theta_B + 36$ ... (4)

* $M_{CD} = \frac{2E(2I)}{12}\left[2\theta_C + 0 + 0\right] - 75$

$M_{CD} = \frac{2EI}{3}\theta_C - 75$. .... (5)

* $M_{DC} = \frac{2E(2I)}{12}\left[0 + \theta_C - 0\right] + 75$

$M_{DC} = \frac{EI}{3}\theta_C + 75$ .... (6)

From Joint Equil.

$M_{BA} + M_{BC} = 0$ ... (7)

$M_{CB} + M_{CD} = 0$ ... (8)

* 8 Eqns and 8 unknowns

Eqn (7) $\Rightarrow \frac{2EI}{3}\theta_B + 36 + \frac{2EI}{3}\theta_B + \frac{EI}{3}\theta_C - 36 = 0$.

$\Rightarrow 4\theta_B + \theta_C = 0$. .... (a)

Eqn (8) $\Rightarrow \frac{2EI}{3}\theta_C + \frac{EI}{3}\theta_B + 36 + \frac{2EI}{3}\theta_C - 75 = 0$.

$\Rightarrow \theta_B + 4\theta_C = \frac{117}{EI}$ .... (b)

Solving for θ_B & θ_C, using Eqs (a) & (b) :-

$$\Rightarrow \boxed{\theta_B = -\frac{7.8}{EI}} \quad \text{and} \quad \boxed{\theta_C = +\frac{31.2}{EI}}.$$

Connecting Moments :-

$M_{AB} = -38.6$ kN.m.

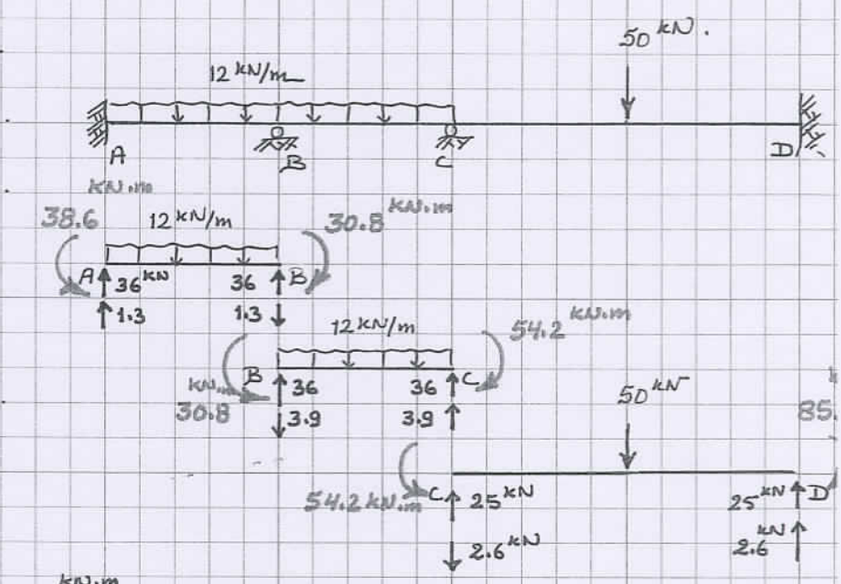
$M_{BA} = 30.8$ kN.m.

$M_{BC} = -30.8$ kN.m.

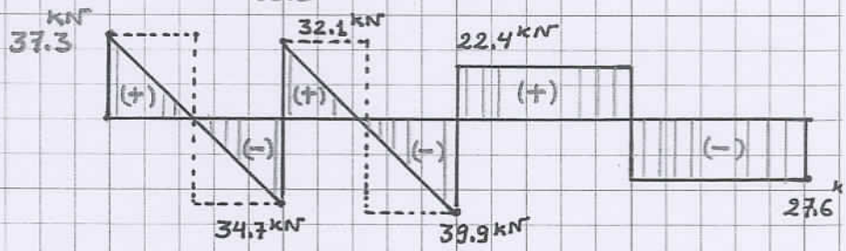
$M_{CB} = 54.2$ kN.m.

$M_{CD} = -54.2$ kN.m.

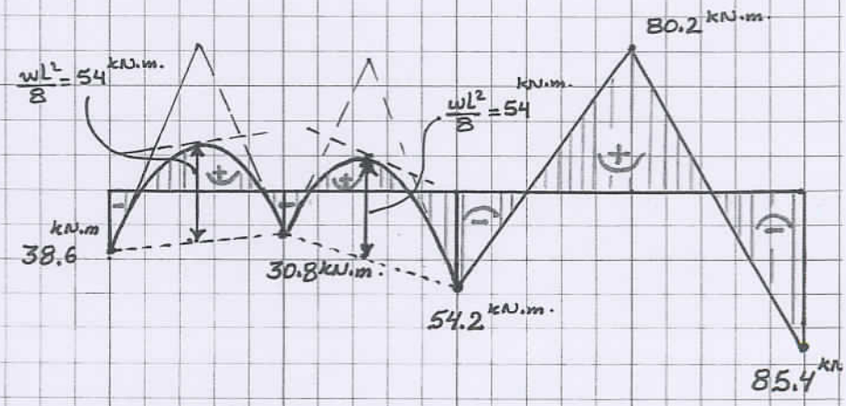
$M_{DC} = 85.4$ kN.m.



S.F.D

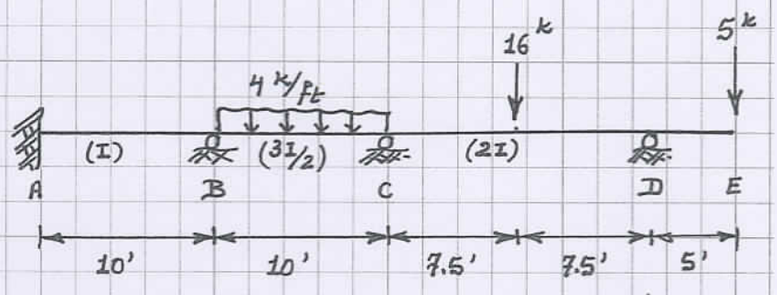


B.M.D

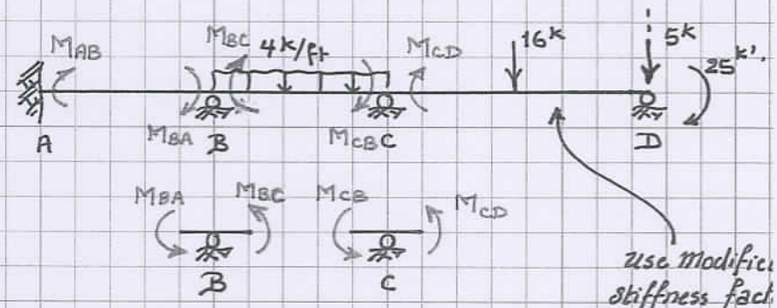


PROBLEM [2]

E = Constant.



Connecting Moments
~assumed clockwise~



Joint Equil.

* $M_{AB} = \frac{2EI}{10}\left[0 + \theta_B - 0\right] + \textcircled{1}$.

$M_{AB} = \frac{EI}{5}.\theta_B$ ---- (1)

* $M_{BA} = \frac{2EI}{10}\left[2\theta_B + 0 - 0\right] + \textcircled{1}$.

$M_{BA} = \frac{2EI}{5}\theta_B$. ------ (2)

* $M_{BC} = \frac{2E(1.5I)}{10}\left[2\theta_B + \theta_C + 0\right] + (-33.33)$

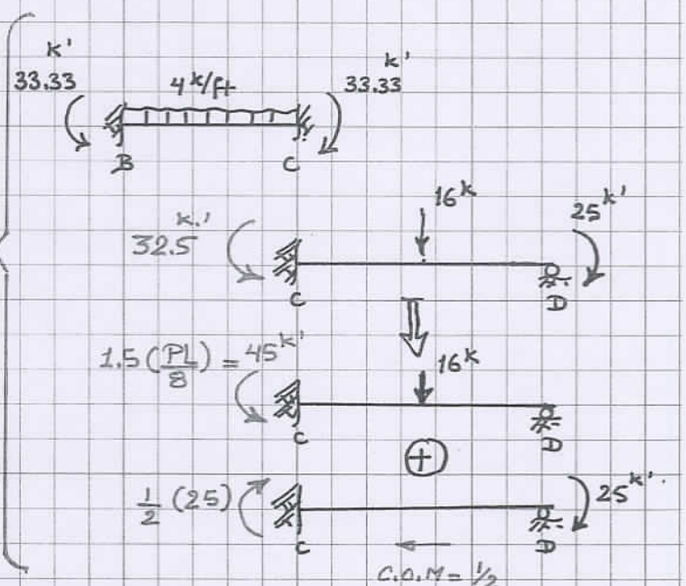
$M_{BC} = \frac{3EI}{5}\theta_B + \frac{3EI}{10}\theta_C - 33.33$ -(3)

* $M_{CB} = \frac{2E(1.5I)}{10}\left[2\theta_C + \theta_B - 0\right] + 33.33$

$M_{CB} = \frac{3EI}{5}\theta_C + \frac{3EI}{10}\theta_B + 33.33$ ...(4)

* $M_{CD} = \frac{3E(2I)}{15}\left[\theta_C + 0\right] - 32.5$.

$M_{CD} = \frac{2EI}{5}\theta_C - 32.5$. ---- (5)



(7 Eqns and 7 unknowns)

From Joint Equilibrium ⇒ $M_{BA} + M_{BC} = \textcircled{1}$ ---(6)

$M_{CB} + M_{CD} = \textcircled{1}$ ---(7).

Substitute in Eqn# (6) :~

$\Rightarrow \frac{2EI}{5}\theta_B + \frac{3EI}{5}\theta_B + \frac{3EI}{10}\theta_C - 33.33 = \textcircled{1} \Rightarrow \frac{10}{3}\theta_B + \theta_C = \frac{111.11}{EI}$ ....(a)

Substitute in Eqn# (7) :~

$\Rightarrow \frac{3EI}{5}\theta_C + \frac{3EI}{10}\theta_B + 33.33 + \frac{2EI}{5}\theta_C - 32.5 = 0 \Rightarrow \theta_B + \frac{10}{3}\theta_C = -\frac{2.777}{EI}$ ...(

Solving for θ_B and θ_C, using Eqns (a) & (b) :-

$$\theta_B = \frac{36.905}{EI} \quad \text{and} \quad \theta_C = -\frac{11.905}{EI}$$

∴ The Connecting Moments are :-

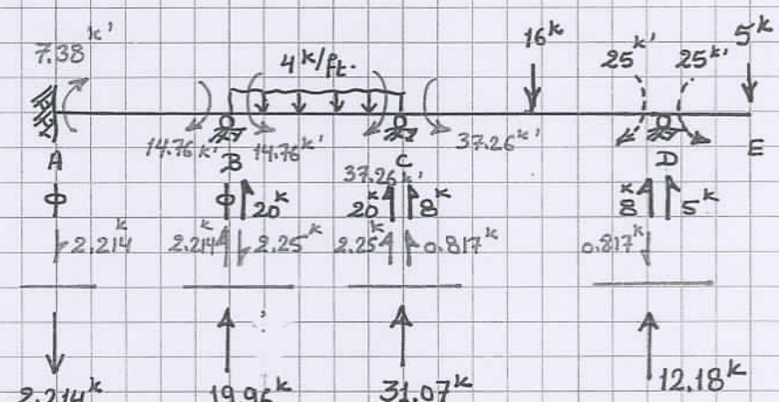
$M_{AB} = +7.38$ k.ft.

$M_{BA} = +14.76$ k-ft.

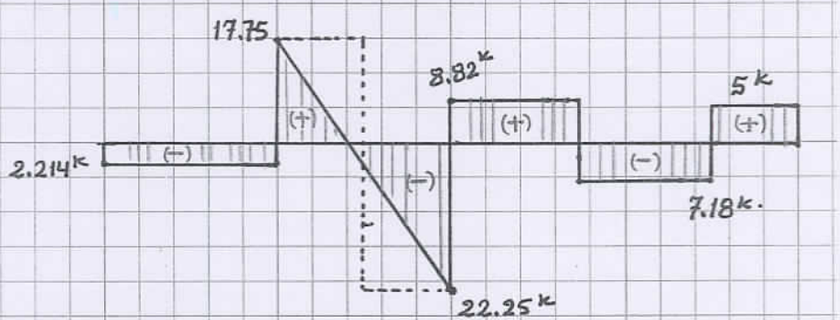
$M_{BC} = -14.76$ k-ft.

$M_{CB} = +37.26$ k-ft.

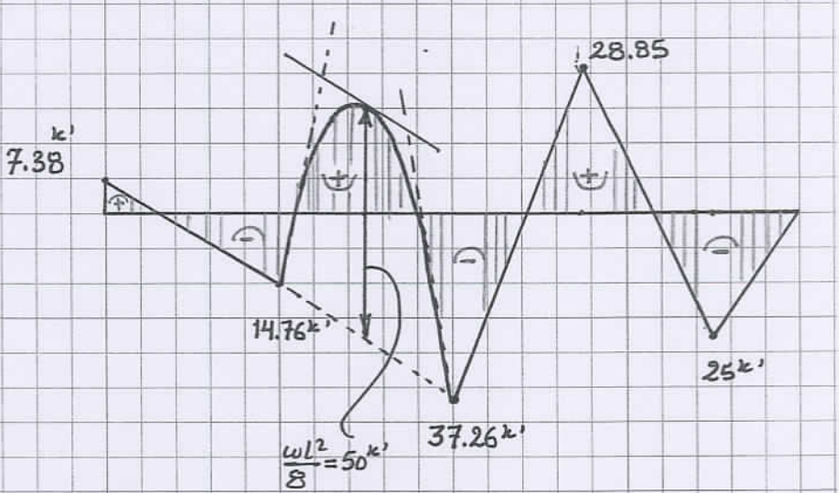
$M_{CD} = -37.26$ k-ft.



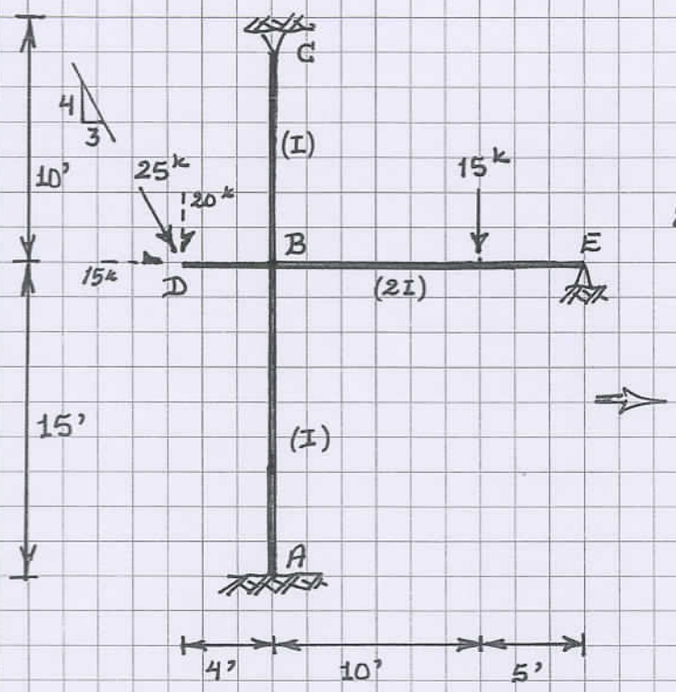
S.F.D

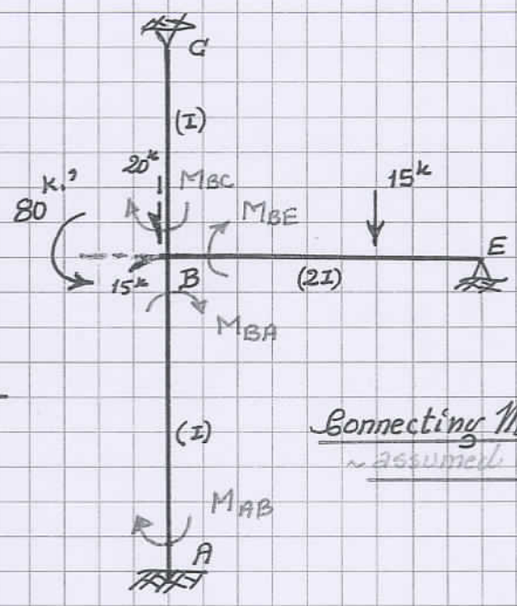


B.M.D.



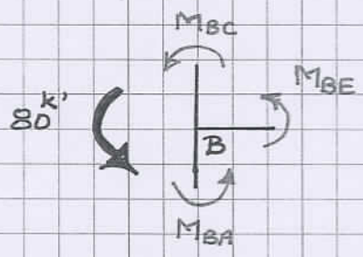
PROBLEM [3].



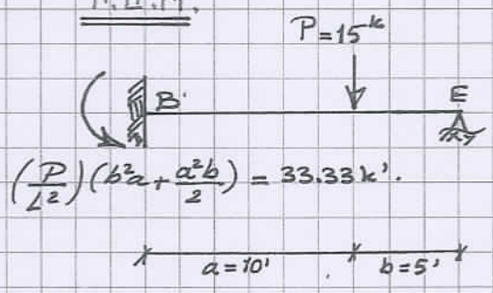


Connecting Moments ~ assumed Clockwise

Joint Equil.



F.E.M.



$$\left(\frac{P}{L^2}\right)\left(b^2a+\frac{a^2b}{2}\right) = 33.33\ k'.$$

* $M_{AB} = \frac{2EI}{15}\left[0+\theta_B+0\right]+0 = \frac{2EI}{15}\theta_B$. ----- (1)

* $M_{BA} = \frac{2EI}{15}\left[2\theta_B+0+0\right]+0 = \frac{4EI}{15}\theta_B$. ----- (2)

* $M_{BE} = \frac{3E(2I)}{15}\left[\theta_B+0\right]+(-33.33) = \frac{6EI}{15}\theta_B - 33.33$. --- (3)

* $M_{BC} = \frac{3EI}{10}\left[\theta_B+0\right]+0 = \frac{3EI}{10}\theta_B$. ----- (4).

Joint Equil. (B) $\Rightarrow$ $M_{BA}+M_{BE}+M_{BC}+80 = 0$. --- (5).

5 Eqns, and 5 Unknowns. $\Rightarrow$ Substitute in (5).

$$\Rightarrow \frac{4EI}{15}\theta_B + \frac{6EI}{15}\theta_B - 33.33 + \frac{3EI}{10}\theta_B + 80 = 0.$$

$$\Rightarrow 0.9667\,EI.\theta_B = -46.667 \Rightarrow \theta_B = -\frac{48.28}{EI}.$$

USE FOR ANSWER

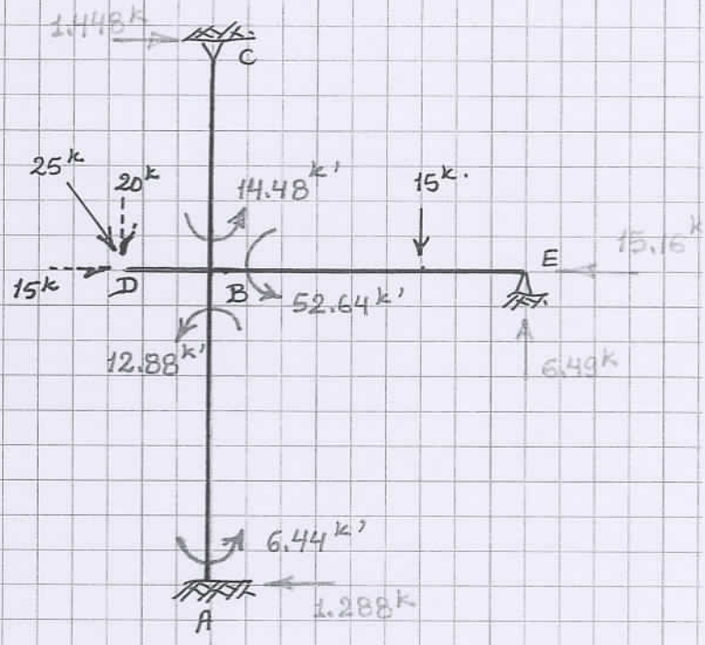
⇒ Connecting Moments :-

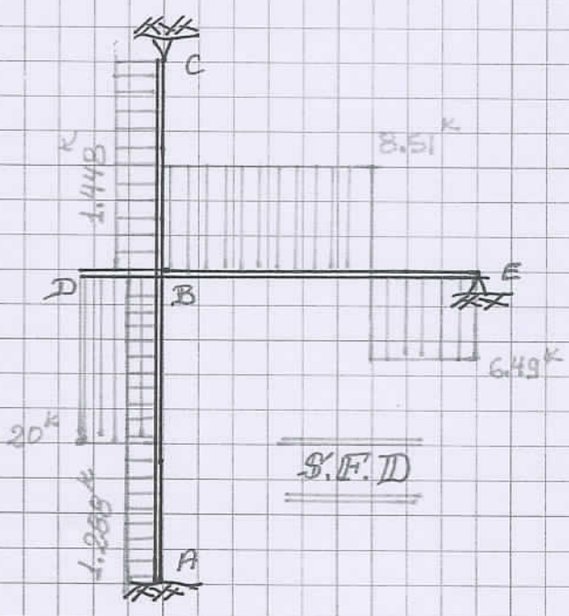
$M_{AB} = -6.44$ k.ft.

$M_{BA} = -12.88$ k.ft.

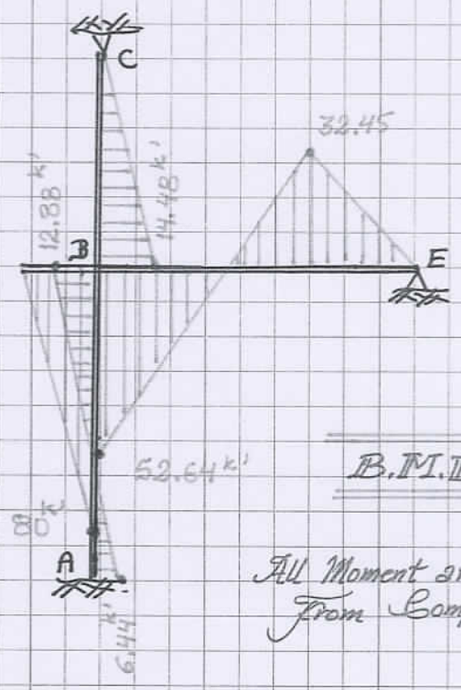
$M_{BE} = -52.64$ k.ft.

$M_{BC} = -14.48$ k.ft.





S.F.D



B.M.D.

All Moment are Plotted From Comp. Side.